SYMBOL	CONFIGURATION	ANGLE, DEG
0	SMALLER	160
0	SMALLER	80
▽	SMALLER	10
♦	LARGER	160
\triangledown	LARGER	80
•	LARGER	10

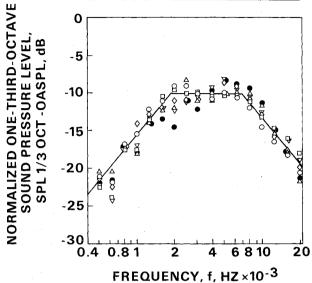


Fig. 3 Normalized spectra for semicylinder reversers.

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Unique Characteristics of Exhaust-Plume Interference

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Nomenclature

A = cross-sectional area

 $C_D = \text{drag coefficient}, D/(q_0 A_m)$

 C_{Dp} = pressure drag coefficient, $(1/A_m) \int_{Am}^{Ab} C_p dA$

 $C_p = \text{pressure coefficient}, (p - p_0)/q_0$

D = nozzle external drag

M = Mach number

 $NPR = \text{nozzle pressure ratio}, P_j/p_0$

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 P_j = nozzle-exhaust total pressure

p = static pressure

q = dynamic pressure

Subscripts

b = nozzle end

m = nozzle maximum

o = freestream

Introduction

AERODYNAMIC interference is a term often used to describe the interactions between closely spaced solid bodies. But interference between solid bodies and plumes also occurs. For example, jet-engine exhaust plumes can significantly affect aerodynamic forces on nearby airplane structure. In fact, this frequently happens. Studies have shown that airplane drag, lift, and moments can vary if plume conditions change.^{1–8}

What complicates the prediction of plume interference is that plumes, unlike solid bodies, create entrainment effects as a result of viscous shear, or mixing, with the surrounding flow. Yet plumes are also obstacles to the surrounding flow, as are solid bodies. Therefore, plume interference can be depicted as being a composite of two constituents, plume entrainment and plume shape, which iteracts with airplane flowfields.

Objectives and Methodology

Researchers using various techniques have examined the fundamental case of jet interference on isolated exhaust nozzles. 9-12 Changes in plume conditions, such as different pressure ratios, can significantly change nozzle external drag. These interference effects are more noticeable during subsonic and transonic flight than during supersonic flight. At supersonic speeds, disturbances can transmit upstream, toward the nozzle, only through the relatively small subsonic portion of the nozzle boundary layer.

Exhaust plumes are represented in some analyses as being solid bodies, often cylinders. ¹³⁻¹⁶ This is obviously a simplification of the true situation; nevertheless, it is an approach sometimes used for pragmatic reasons. So that such approaches may be evaluated, an objective of this study was to compare, experimentally, flowing jets with

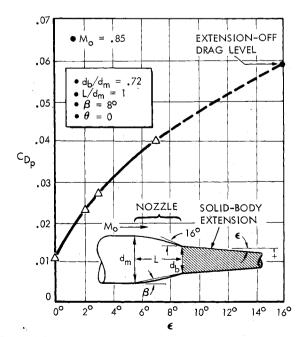


Fig. 1 Effects of solid-body plume simulators on nozzle pressure drag.

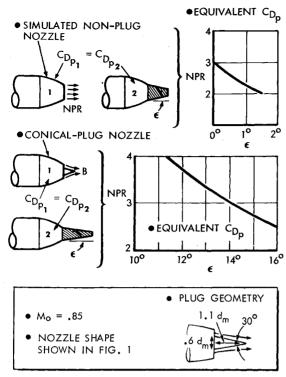


Fig. 2 Equivalent drag crossplots.

solid-body plume simulation. Another objective was to examine plume interference effects associated with changes in nozzle shape as well as changes in plume condition.

Experiments were conducted in a freejet wind-tunnel facility at the Fort Worth operation of the Convair Aerospace Division. Descriptions of this facility have been published.¹⁷ All the nozzle models tested are bodies of

revolution, 3 in. in maximum external diameter. These models have circular-arc external contours and their internal flow areas become minimal at their exit plane. Each nozzle, connected to a cylindrical upstream-support pipe, is subjected to changes in freestream Mach number and nozzle pressure ratio (exhaust total pressure divided by freestream static pressure). External surface flow at the nozzle connect station has a turbulent boundary layer. its height being approximately 17% of the pipe diameter. Rows of static-pressure orifices comprise the model external instrumentation. The pressure readings from these orifices are plotted vs aft-facing projected area and then integrated to obtain nozzle pressure drag. In addition, total-pressure tubes located inside each nozzle measure nozzle pressure ratio. All instrumentation is of the steady-state type, connected to a scanivalved transducer located outside the test section.

Results and Analysis

Solid-body plume simulation

Several solid-body extensions, or "plume simulators," were alternately attached to a nozzle, as shown in the insert sketch on Fig. 1. For simplicity, these extensions were conical shapes, with the half angle of each denoted as ϵ . Nozzle pressure drag, plotted in Fig. 1, increases several fold as the extensions proceed from being cylindrical ϵ = 0. through the conics tested, up to $\epsilon = 7^{\circ}$. The nozzle drag increases because, as the solid extensions become more tapered, less flow recompression occurs near the end of the nozzle. For example, nozzle drag at $\epsilon = 1.5^{\circ}$ is twice that at $\epsilon = 0$. Thus, seemingly small changes in extension shape can cause large changes in the upstream flowfield. The dashed line extending from $\epsilon = 7^{\circ}$ to 16° in Fig. 1 is an extrapolation to the drag level that occurs with no extension attached and with no exhaust flow. Note that the end of the nozzle is at a 16° angle; thus extrapolating

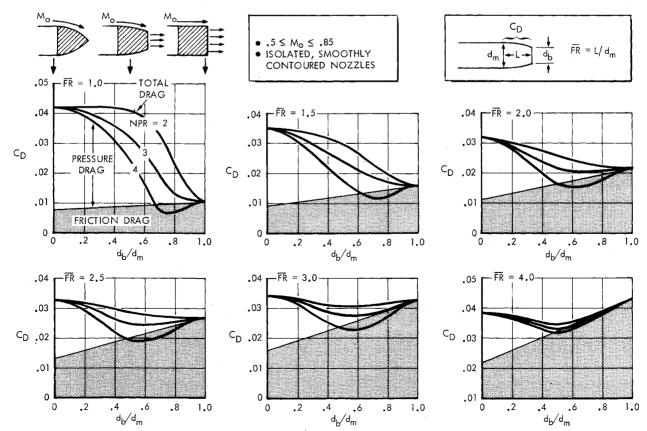


Fig. 3 Nozzle drag correlation.

to an ϵ corresponding to this value for the jetoff condition seems within reason, at least for the sake of example.

Next, the solid extensions were replaced with flowing jets. Two types of plumes were tested: one representing a nonplug-nozzle exhaust plume, the other an exhaust plume surrounding a conical centerbody plug. Again, nozzle pressure drag was measured. But, rather than varying ϵ , data were taken at several nozzle pressure ratios from jet off to 4. Forces occurring on the centerbody plug itself were not measured in these experiments.

The jeton and the solid-extension results were combined to produce the crossplots shown in Fig. 2. The purpose here is to ascertain what solid-extension shapes produce nozzle drag levels equivalent to those at jeton conditions. For the nonplug nozzle, one might expect the plume shape to be cylindrical at ideally expanded, just-choked conditions (NPR \approx 2); however, it is especially significant in Fig. 2 that at this NPR a solid body narrower than a cylinder is required to simulate jet interference; the "effective" plume shape is cylindrical only when the plume billows at the underexpanded NPR = 3 condition. A conic solid body having a 1.5° half angle produces the correct interference effects at NPR = 2 for the nonplug nozzle, but note that at NPR's from 2 to 2.5, no solid body can simulate the same interference on the plug-type nozzle; in other words the plug nozzle tested has more external drag (excluding plug forces) at NPR's from 2 to 2.5 than at jetoff conditions. Apparently, plume entrainment, a unique characteristic of exhaust-plume interference, plays an important role in producing these interference

The solid-body shapes required to generate equivalent plume-interference effects on the plug nozzle at NPR's greater than 2.5 are more tapered than those appropriate for the nonplug nozzle. This is an expected result because the plume attempts to follow the tapered centerbody-plug contour.

To retain proper perspective, it should be emphasized that these results are linked to specific nozzles and cannot be expected to be independent, at least quantitatively, of aftbody/nozzle geometry.

Nozzle drag correlation

Nozzle drag correlations are desirable for establishing general overviews of nozzle behavior as engine power setting and nozzle geometry vary. Drag data from this study plus other recent Convair studies¹⁷⁻¹⁹ were compiled along with other empirical data^{9,13,20,21} to generate the curves presented in Fig. 3. The two geometric parameters used here are nozzle fineness ratio (nozzle length divided by nozzle maximum diameter) and diameter ratio (d_b/d_m) . Only smoothly contoured nozzle shapes, e.g., circular arc or parabolic, having thin trailing edges are considered here; nozzles having abrupt contour changes and/or thick bases usually exhibit a different drag behavior.

Total nozzle drag and its components, pressure drag and friction drag, are shown in the plots of Fig. 3. Notice that the nozzle shape at $d_b/d_m=0$ reaches an imaginary boundary condition where no exhaust orifice exists; here the pressure drag is large compared to the friction drag. The opposite occurs at $d_b/d_m=1$, where the larger surface area causes friction drag to increase considerably, but the cylindrical shape prevents pressure drag from occurring

Increases in fineness ratio result in decreased pressure drag but increased friction drag. In the limit, pressure drag approaches zero as fineness ratio approaches infinity; however, in practical terms, pressure drag for a typical range of nozzle positions $(0.4 < d_b/d_m < 1)$ rapidly approaches a negligible level when a fineness ratio of 4 is reached. Changes in exhaust area and/or nozzle pressure ratio noticeably affect total drag at fineness ratios less

than 4; at 4, plume-interference effects are not large since the exhaust plume is somewhat remote with respect to the overall nozzle surface. Fineness ratios from 1.5 to 2 apparently produce the least amount of pressure-plus-friction drag.

Increases in nozzle pressure ratio from an NPR = 2 to 3 or 4 result in less nozzle drag at fineness ratios between 1 and 4. However, nozzle thrust efficiency is reduced for convergent nozzles at underexpanded-plume conditions. Nozzle selection criteria must therefore involve thrustminus-drag tradeoffs to account for these effects.

Conclusions

Exhaust-plume interference on nozzle drag is a function of plume entrainment as well as plume shape. In some situations, certain solid bodies—with narrower shapes than might be expected—can be used in place of plumes to simulate plume interference on drag. However, there are also some nozzle geometry and pressure-ratio conditions in which solid bodies cannot simulate exhaust-plume interference. Therefore, one must carefully examine situations involving solid-body plume simulation.

The drag of isolated, smoothly contoured nozzles with typical exhaust conditions is significantly affected by plume interference at nozzle length-to-diameter ratios below 4. Of the nozzle configurations considered, length-to-diameter ratios from 1.5 to 2 produce the least amount of pressure-plus-friction drag.

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Angular Momentum and the Aircraft-Store Separation Problem

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Nomenclature

inner body K_{1},K_{2} = complex constants defined by the initial conditions m = total configuration mass m_{1},m_{2} = masses of outer and inner bodies M_{q} = Magnus moment stability derivative M_{q} + M_{α} = pitch damping moment stability derivative M_{xA},M_{yA},M_{zA} = moments about the X_{A},Y_{A},Z_{A} axes M_{xB},M_{yB},M_{zB} = moments about the X_{B},Y_{B},Z_{B} axes = pitching moment stability derivative M_{q} = pitching moment stability derivative M_{q} = angular velocity components resolved along the X_{q} + X_{q} - X_{q} axes X_{q} = time X_{q} = body linear velocity components resolved along X_{q} + X_{q} - X_{q} axes X_{q} = right handed coordinate system when X_{q} the missile axis of symmetry X_{q} = distance from c.g. of outer body to total configuration c.g.		
$F_{xB}, F_{yB}, F_{zB} = \text{forces along the } X_B, Y_B, Z_B \text{ axes}$ $I_{x}I_y = \text{rolling and pitching moments of inertia}$ $I_{x_2}I_{y_2} = \text{rolling moment of inertia of outer body are inner body}$ $I_{y1}I_{y_2} = \text{pitching moment of inertia of outer body are inner body}$ $K_1, K_2 = \text{complex constants defined by the initial conditions}$ $m = \text{total configuration mass}$ $m_{1}, m_{2} = \text{masses of outer and inner bodies}$ $M_{p\alpha} = \text{Magnus moment stability derivative}$ $M_{q} + M_{\alpha} = \text{pitch damping moment stability derivative}$ $m_{xA}M_{yA}M_{yB}M_{zB} = \text{moments about the } X_A, Y_A, Z_A \text{ axes}$ $m_{p,q,r} = \text{pitching moment stability derivative}$ $m_{p,q,r} = \text{moments about the } X_B, Y_B, Z_B \text{ axes}$ $m_{p,q,r} = \text{pitching moment stability derivative}$ $m_{p,q,r} = \text{angular velocity components resolved along } X_B, Y_B, Z_B, X_B, X_B, X_B, X_B, X_B, X_B, X_B, X$	F_{rA}, F_{vA}, F_{zA}	= forces along the X_A , Y_A , Z_A axes
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the missile axis of symmetry x_1 = distance from c.g. of outer body to total configuration c.g. x_2 = distance from c.g. of inner body to total configuration c.g. Z_{α} = normal force stability derivative	X, Y, Z	
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configuration c.g. Z_{α} = normal force stability derivative		configuration c.g.
Z_{α} = normal force stability derivative	x_2	= distance from c.g. of inner body to total
		configuration c.g.
α = Complex angle of attack	Z_{lpha}	= normal force stability derivative
	α	= Complex angle of attack

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= dynamic damping factors

 $\omega_1.\omega_2$

= nutation and precession frequencies

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cripts

A	= aeroballistic axis system
B	= body fixed axis system
1	= outer body
2	= inner body

Introduction

AIRCRAFT-store separation is a serious consideration for both weapon and aircraft designers. The weapon designer wants to avoid the large release disturbance since it affects the weapons accuracy. The aircraft designer wants to avoid the large release disturbance since it is dangerous to the pilot and can result in damage to the aircraft.

There have been various methods of improving weapon separation characteristics but none has been entirely satisfactory since they are based largely upon trial and error. Present systems are rather erratic in their separation characteristics due to variation in store configurations.

In retrospect, it is quite unlikely that a store separation problem existed when aircraft flew at 250 knots and dropped 2000 lb bombs due to the inertia loads being much greater than the aerodynamic loads. The bombs were aerodynamically inert at release.

However, aircraft speed has increased considerably. The aerodynamic loads increased with velocity squared and store separation became a problem. Weapon accuracy was reduced and aircraft damage was sustained.

Aircraft can now carry high density, externally stored weapons supersonically. It would also be advantageous to launch these weapons supersonically. The aerodynamic loads will be much larger and if large release disturbances are encountered, the damage to the aircraft can be severe.

One method of alleviating the store separation is to gyroscopically increase the weapon's inertial forces by inserting a spinning fly wheel. Admittedly, this is an added complication but it may be required to solve the problem. It is the purpose of this paper to present the mechanics of the internally stabilized weapon.

Rigid Body Dynamics

From the linear theory of missile dynamics it can be shown that an approximate solution for the angle of attack of a spinning, symmetric, rigid body (neglecting damping) is

$$\alpha = K_1 e^{(i\omega_1 t)} + K_2 e^{(i\omega_2 t)} \tag{1}$$

If $\alpha_0 = 0$ at t = 0 then it may be shown that for an initial angular rate the maximum angle of attack is

$$|\alpha_{\text{MAX}}| = 2|\dot{\alpha}_0|/[(p^2I_x^2/I_y^2) - 4M_\alpha/I_y]^{1/2}$$
 (2)

Consequently, for statically stable configurations, the maximum angle of attack due to an angular rate may be reduced by increasing the angular momentum or by increasing the restoring moment. It is suggested that external stores be designed in accordance with Eq. (2).

The restoring moment for an external store is usually small due to the design conforming to space limitations. However its angular momentum may be increased considerably.

Two methods of increasing the angular momentum of external stores are readily apparent. The first would require spinning the store on the rack. This method would necessitate a complicated rack design which would probably not work for all stores. Moreover, the stores would be released with a high spin rate and be subject to Magnus instability.

A second method for increasing the angular momentum of an external store is to spin an internally mounted fly wheel. This method would work equally well for all types